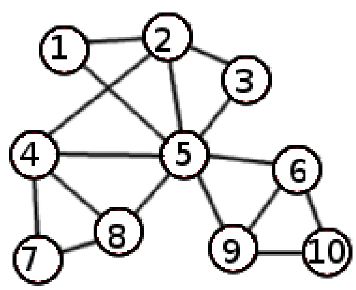
Graph Theory Homework 3
Due: 10 June 2019 at 3:59pm as a PDF on Submitty
v1.1: Updated 06 June 2019

1. For a simple connected graph G and arbitrarily-selected vertices $u, v \in V(G)$, the number of edge disjoint u, v-paths is x. What can we infer about the connectivity $\kappa(G)$ and edge-connectivity $\kappa'(G)$ of G from the preceding statements? Justify your responses.

2. We have written a function that identifies a maximum set of edge-disjoint u, v-paths in a graph G in linear time. To use our function, we pass a graph G with two vertices u, v and get returned edge-disjoint u, v-paths as a set of paths defined by vertices and edges. E.g., calling `getAllPaths(`G, u, v`)` would possibly return $\big\{\{u, e_1, v_1, e_2, v_2, e_3, v\}, \{u, e_4, v_1, e_5, v_3, e_6, v\}\big\}$.

   How can we use this function to determine the edge connectivity $\kappa'(G)$ of a graph G in polynomial time? Show pseudocode or give a detailed description of your approach.

3. Is a closed ear decomposition of the below graph possible? What about an open ear decomposition? Draw one for each if possible. What does this demonstrate about its connectivity and edge-connectivity?



4. Consider tree T with perfect matching M. Prove that M must be unique. (**v1.1**: Don't need to use induction)

5. Consider a biconnectivity decomposition. Show that for graph G, $\forall v \in V(G) : d(v)$ is even iff for every maximal biconnected component $B_i \in G$, $\forall u \in V(B_i) : d(u)$ is even. Hint: one direction is trivial and the other might require a bit of induction.